THE EFFECT OF A MAGNETIC FIELD ON THE RESISTANCE BETWEEN ELECTRODES IN A CURRENT-CONDUCTING CHANNEL WITH AN ANISOTROPIC MEDIUM

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An electrical network is used to model the function of the resistance between electrodes of various shapes in a current-conducting channel with an anisotropic medium.

One of the concerns of electromagnetic thermal physics is the study of heat- and mass-transfer processes and the effect of electric and magnetic fields on these processes. Such an investigation is generally based on the simultaneous solution of the equations of hydrodynamics, energy, and diffusion. The distributions of velocity, temperature, and concentration found from these solutions enable us to determine the necessary coefficients of friction, heat transfer and mass transfer, as well as the nature of the influence on these coefficients as exerted by the factors of interest to us. The difficulties encountered in an analytical investigation into the phenomena of transfer increase as the influence of external electromagnetic fields becomes evident.

For plasma flows with low velocities (compared to the local speed of sound) the solution of the thermophysical problems is substantially simplified. If we do not bother in this case with the temperature variation in the physical characteristics, it becomes possible to isolate the independent electromagnetic parts from the general system of equations, as well as to separate the solution of the hydrodynamic problem from the thermal problem [1].

This is the problem which we encounter in examining the effect of a magnetic field on the current distribution in an arc discharge in a low vacuum, when the plasma occupies the entire lateral cross section of the channel (a magnetic ring arc). In a channel of circular cross section, where the Hall currents close on

TABLE 1. The Value of Collision Parameter $\omega_e\tau_e$ as a Function of the Change in Resistance within the Network Circuits

R'	$\frac{r}{10}$	$\frac{r}{9}$	$\frac{r}{8}$	$\frac{r}{7}$	$\frac{r}{6}$	$\frac{r}{5}$	$\frac{r}{4}$	$\frac{r}{3}$	$\frac{r}{2}$	r
$\omega_e\tau_e$	0	0,11	0,19	0,21	0,26	1	1,25	1,52	2	3

TABLE 2. R/R_0 as a Function of $\omega_e\tau_e$ for Electrodes Simple and Complex in Shape

Electrode shape	$\omega_e\tau_e$								
	0,1	0,19	0,21	0,26	1	1,25	1,52	2	3
Simple	1,06	1,22	1,36	1,54	1,81	2,24	2,86	4,0	7,1
Complex	1,01	1,21	1,38	1,59	1,86	2,24	2,87	4,1	7,36

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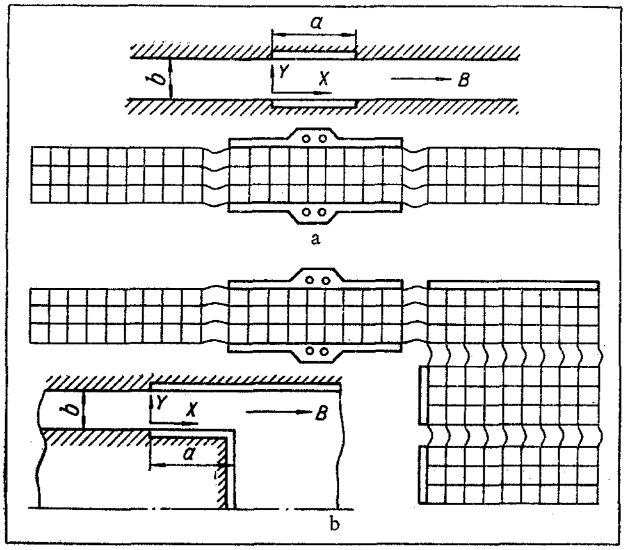

Fig. 1. Simple (a) and compound (b) shapes of electrodes and resistance circuits of modeling regions.

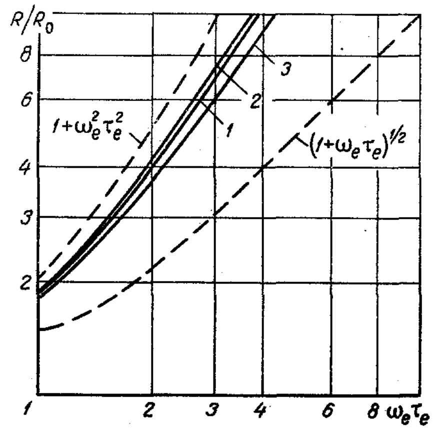

Fig. 2. Ratio of resistances between electrodes in the presence and absence of a magnetic field and without it.

themselves, Ohm's law has the simple form

$$\bar{j} = \sigma \bar{E}, \tag{1}$$

but since $\omega_e \tau_e > 1$, we find anisotropy of conductivity and the electrical conductivity σ is a tensor whose matrix in a coordinate system with an X-axis parallel to the magnetic field [3] is equal to

$$\sigma = \frac{\sigma_0}{1 + \omega_e^2 \tau_e^2} \begin{pmatrix} 1 + \omega_e^2 \tau_e^2 & 0 & 0 \\ 0 & 1 & -\omega_e \tau_e \\ 0 & \omega_e \tau_e & 1 \end{pmatrix}. \tag{2}$$

Under these conditions, the solution for the problem of determining the local and integral characteristics of the electric field is found by isotropically deforming the space along the coordinate axes. In the isotropic space derived in this manner, the problems are solved in the usual fashion [4,5].

In this paper we will investigate the effect of current-distribution edge effects on the resistance between electrodes with the various configurations that are frequently encountered in plasma generators.

The boundary-value problem of current distribution encountered with the use of electrodes more complex in shape can be solved analytically, but the solution will be complex and laborious. It should be borne in mind that the problem is idealized, does not require a high degree of accuracy, and that it is necessary that we find the general integral characteristic – the ratio of the voltages at the electrodes to the total current.

These circumstances enable us to employ a simulation method to the solution, namely, the method of the electrical network, in which the continuous medium is replaced by "equivalent" concentrated elements.

The basic methods of resistance-network design are covered in [6-8].

To solve the two-dimensional field-theory problem of determining the resistance between the electrodes as a function of the collision parameter $\omega_e \tau_e$ we put together a resistance network of the following form. Four rows of thin constantan conductors are positioned longitudinally along a nonconducting rectangular plate, and 10 rows of such conductors are positioned in the transverse direction. Each row consists of 10 conductors. The nodes at which the conductors cross are soldered. To satisfy the boundary conditions, the individual plates are connected by means of copper conductors. The total resistance between two equipotential field boundaries was measured with a double-bridge UPIP-60 instrument. Figure 1a shows the geometrically simple shape of the electrodes and the simulation network. For the case of isotropic conductivity, with 10 conductors laid out in two directions of the field, we found that there is no need to hook up additional sections, since the magnitude of the total resistance (accurate to within 0.1%) is not altered when these are connected.

To establish conductivity anisotropy in the direction of the field parallel to B (X-axis) the number of conductors was retained, while in the direction perpendicular to B (Y-axis) the number of conductors was gradually reduced. This corresponds to the relationships

$$\sigma_X = \sigma_0, \quad \sigma_Y = \frac{\sigma_0}{1 + \omega_e^2 \tau_e^2}. \tag{3}$$

The increase in the network resistance in the direction of the Y-axis corresponds to values of $\omega_e \tau_e$ as shown in Table 1, where R' is the total resistance between the nodes of the network and r is the resistance of a single constantan conductor between the nodes.

Table 2 shows the experimental data with regard to R/R_0 as a function of $\omega_e \tau_e$ for electrode shapes whose geometric dimensions and simulation network are shown in Fig. 1a. For R_0 we assume the total resistance of one rectangular portion of the network between two equipotential field boundaries. For this same electrode shape we have established that in the presence of isotropic conductivity there is an edge effect associated with the existence of a conducting zone on the two sides of the electrodes, but that its magnitude is small. The table also shows experimental data for electrodes of more complex shape, whose geometric dimensions and simulation network are shown in Fig. 1b.

On the basis of the data in Table 2, Fig. 2 shows graphically the resistance in the presence of a magnetic field as a ratio of the resistance for the same geometric electrode configuration, but with $\omega_e \tau_e = 0$ (curves 1 and 2), the ratio $a/b = 2.7$. For comparison, here we have also plotted the curves $1 + \omega_e^2 \tau_e^2)^{1/2}$ and $(1 + \omega_e^2 \tau_e^2)$, as well as curve 3 from the data of [4], for a ratio $a/b = 1$, for the electrode shape indicated in Fig. 1a.

These results permit us to draw the following conclusions. A change in the total resistance between electrodes of various shapes in the presence of a magnetic field and with anisotropy of conductivity is clearly defined and in agreement with the results of [4]. The existence of current-distribution edge effects is possible if the conducting medium occupies the space on the two sides of electrode, at those segments equal to the width of the electrodes.

NOTATION

$\bar{j}$ is the current density;
$\bar{E}$ is the electric field intensity;
σ is the electric conductivity;
σ_0 is the electric conductivity without magnetic field;
ω_e is the cyclotron frequency of electron rotation;
τ_e is the time of free electron path;
a is the width of electrodes;
b is the distance between electrodes.

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